CheFlet polar bases for astronomical data analysis

Yolanda Jiménez Teja
Txitxo Benítez Lozano
Instituto de Astrofísica de Andalucía (CSIC)
Departamento de Astronomía Extragaláctica
E-mail: yojite@iaa.es

Outline

Motivation

Mathematical background

Visualization

Examples

Applications

- Motivation
- Mathematical background
- Visualization
  - Chebyshev-Fourier basis
  - Non-vanishing wings
- Practical implementation
  - From the continuous to the discrete domain
  - Choice of the scale size
  - Choice of the number of coefficients
    - Elliptical and irregular galaxies
    - Spiral galaxies
- Examples: coefficients and partial reconstruction
- Practical applications

Large photometric redshift surveys

- ALHAMBRA: http://arxiv.org/abs/0806.3021
- PAU: http://fr.arxiv.org/abs/0807.0535

→

- Photometry measurements
- Morphological features

Galaxies decomposition method

- Linear
- Flexible
- Capable of modelling both the bulge and the disk of extended galaxies.

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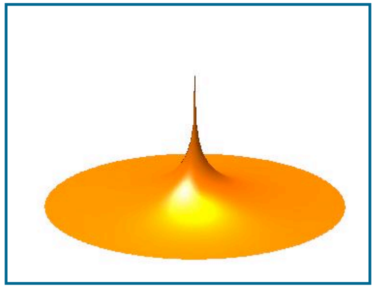
The wings of the basis functions tend to vanish, so the light flux is bounded by the basis:



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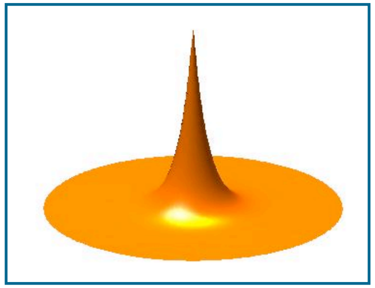
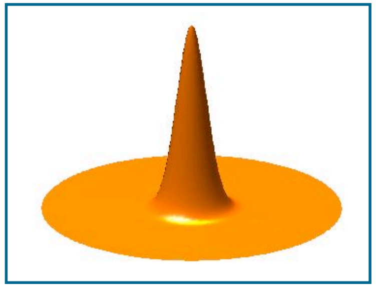
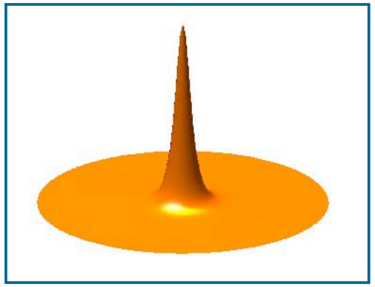


Exponential

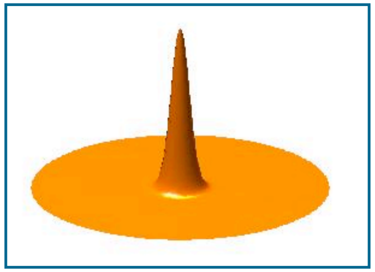


Gaussian

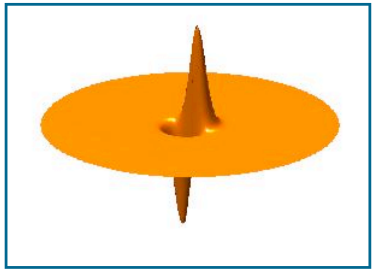


Moffat-Lorentzian

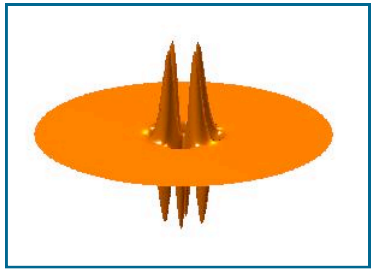
Shapelets



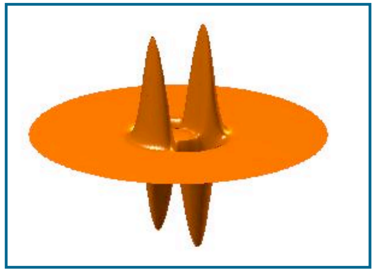
$n = 0, \ m = 0$



$n = 1, \ m = 1$



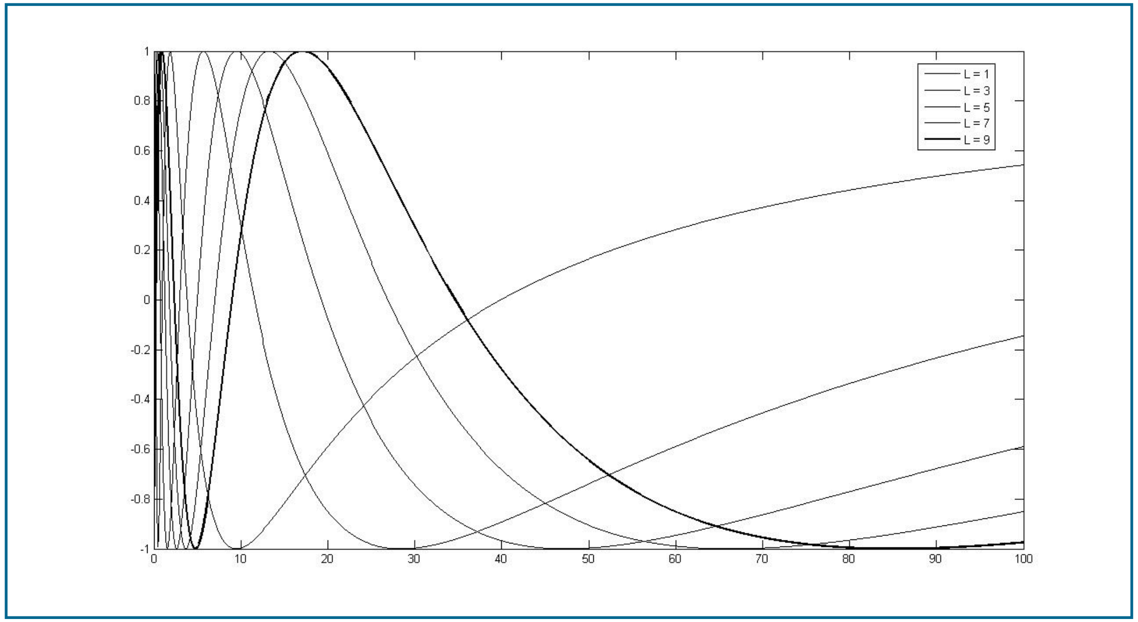
$n = 4, \ m = 4$



$n = 6, \ m = 2$

Chebyshev polynomial: $T_n(r) = \cos(n \cdot \arccos(r))$

Chebyshev rational function: $TL_n(r;L) = \cos\left(n \cdot \arccos\left(\frac{r-L}{r+L}\right)\right)$



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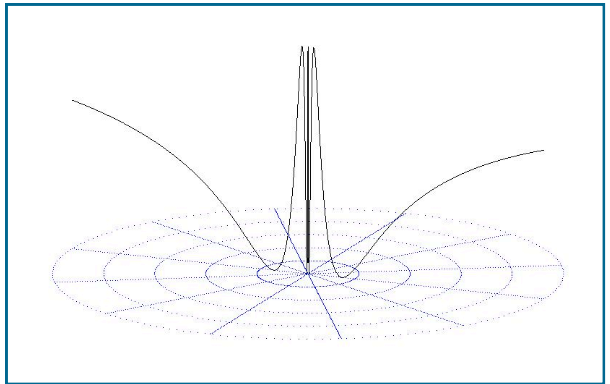
The Cheblet polar basis is separable in *r* and *θ*:

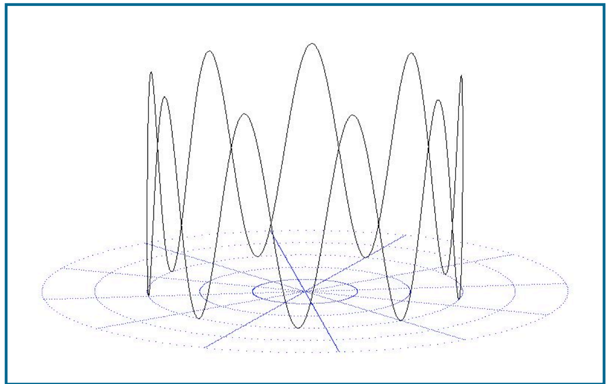
Chebyshev rational functions in *r*

$$TL_{n_1}(r) = T_{n_1}\left(\frac{r-L}{r+L}\right) = \cos\left(n_1 \cdot \arccos\left(\frac{r-L}{r+L}\right)\right)$$



Fourier series in *θ*

$$e^{in_2\theta}$$



$$\left\{\phi_{n_1 n_2}(r,\theta;L)\right\}_{n_1 n_2} = \left\{\frac{C}{\pi} TL_{n_1}(r;L) e^{in_2\theta}\right\}_{n_1 n_2}, \text{ with } C = \begin{cases} 1, & \text{if } n_2 = 0 \\ 2, & \text{if } n_2 > 0 \end{cases}$$

- It is a basis of the Hilbert space $L^2([0,+\infty)\times[-\pi,\pi],\langle\cdot,\cdot\rangle)$, with

$$\langle f,g\rangle = \int_0^{+\infty}\int_{-\pi}^{\pi} f(r,\theta)\overline{g(r,\theta)}\frac{1}{r+L}\sqrt{\frac{L}{r}}\,d\theta\,dr$$

- A smooth function *f* can be decomposed into

$$f(r,\theta)=\frac{C}{2\pi^2}\sum_{n_2=-\infty}^{+\infty}\sum_{n_1=0}^{+\infty} f_{n_1n_2}TL_{n_1}(r)e^{in_2\theta}$$

where

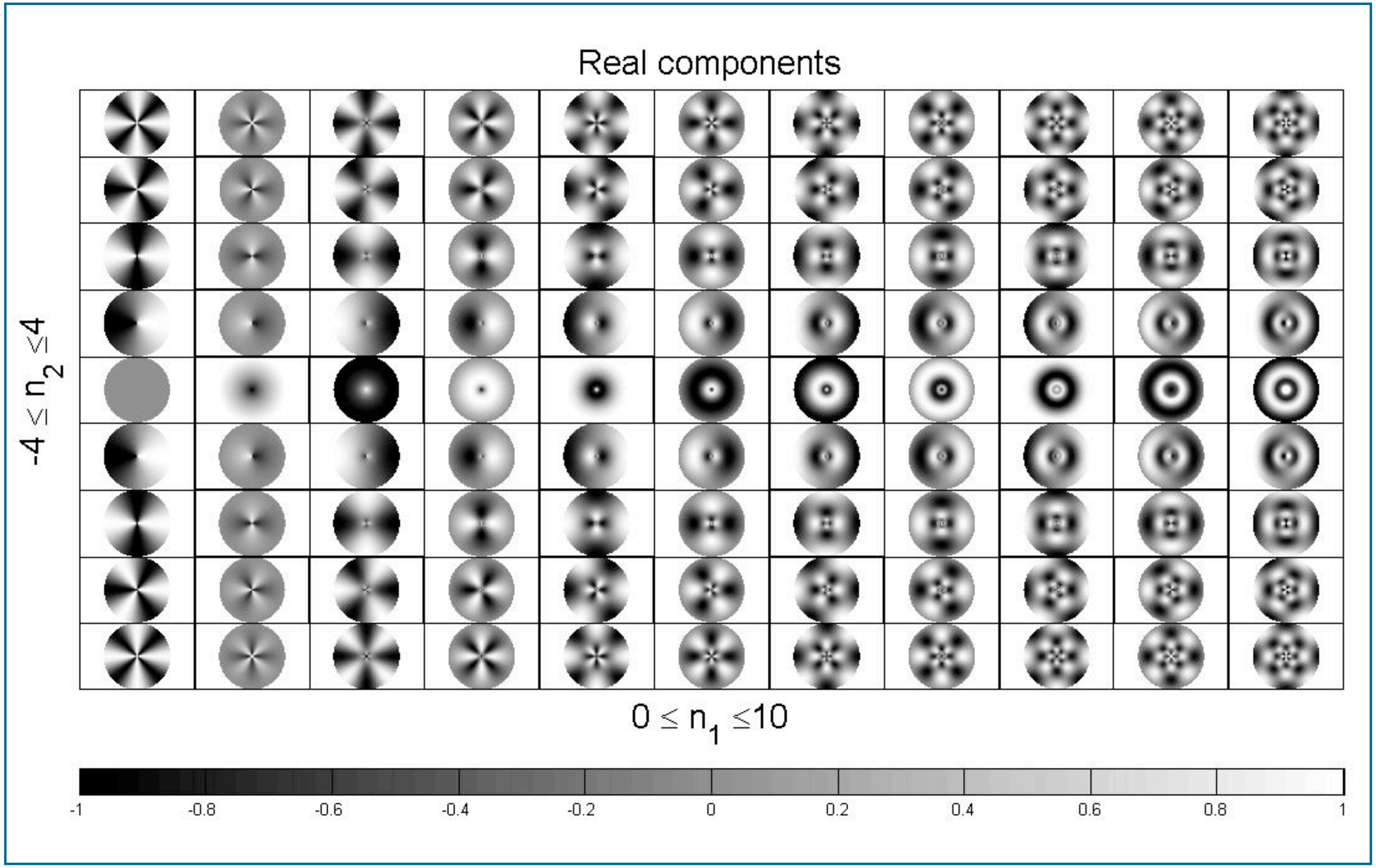
$$f_{n_1n_2}=\frac{C}{2\pi^2}\int_{-\pi}^{\pi}\int_0^{+\infty} f(z,\phi)TL_{n_1}(z)\frac{1}{z+L}\sqrt{\frac{L}{z}}\,e^{-in_2\phi}\,dz\,d\phi$$

- These coefficients show an algebraic decay rate:

$$\left|f_{n_1n_2}\right|\le\frac{A}{\left|n_1\right|\left|n_2\right|^{\frac{p+1}{2}}}$$

where *p* is related to the smoothness of the function *f.*

Cheblet polar basis functions



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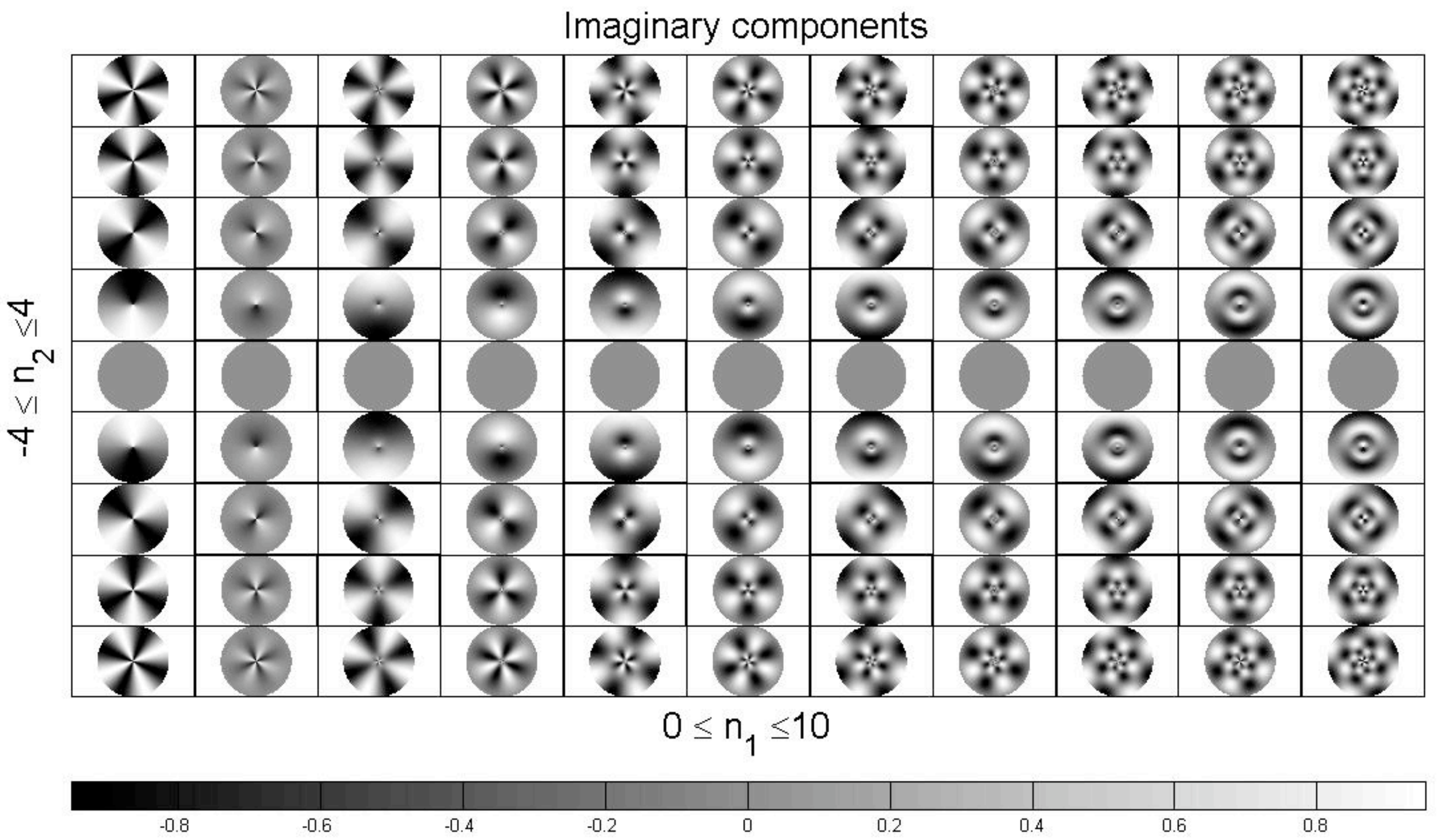

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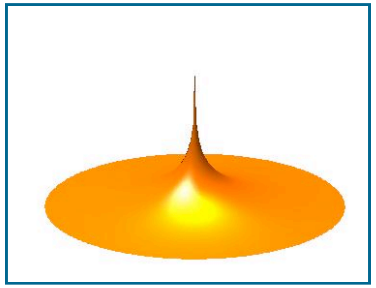

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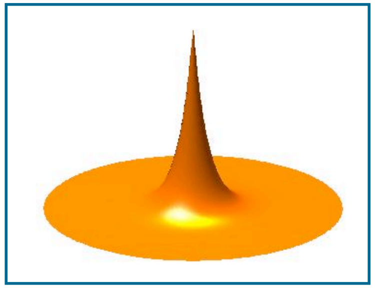

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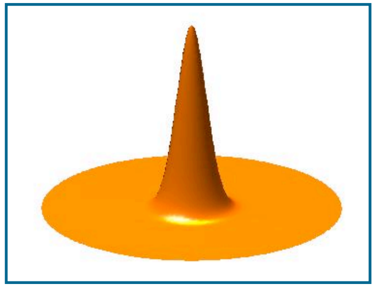

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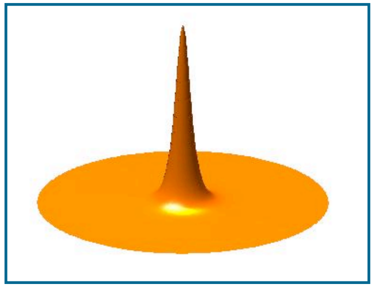

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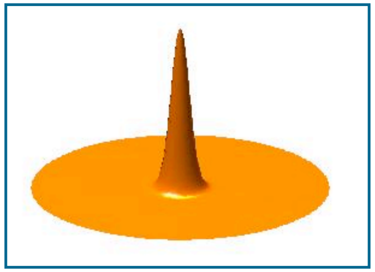

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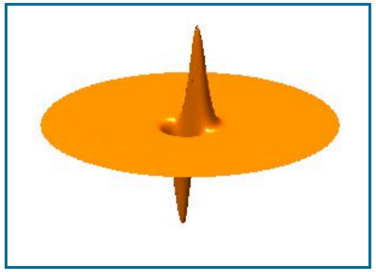

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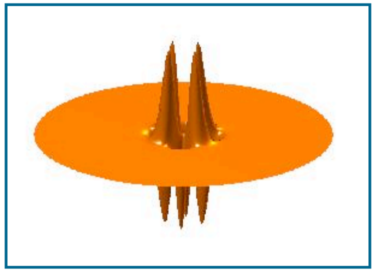

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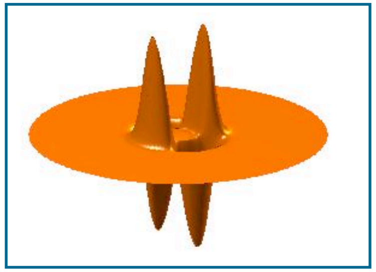
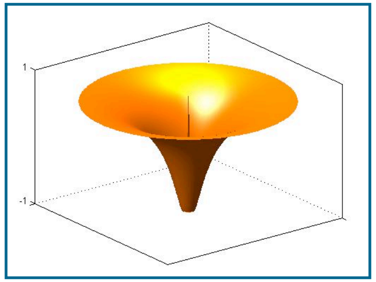
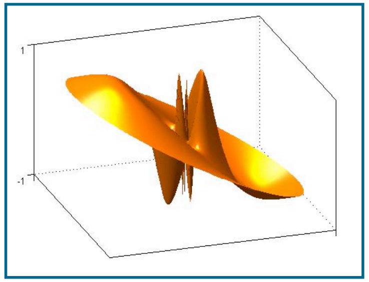


$n = 6, \; m = 2$

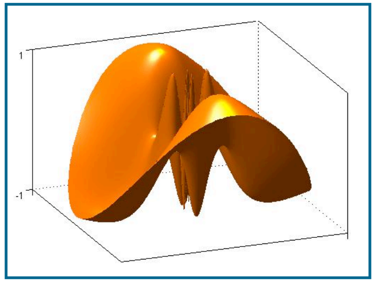
Cheblet basis



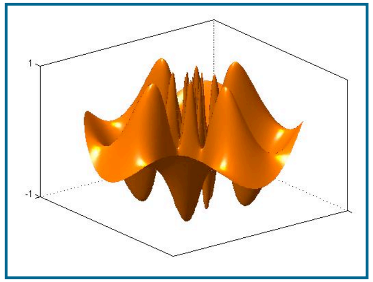
$n_1 = 2, \; n_2 = 0$



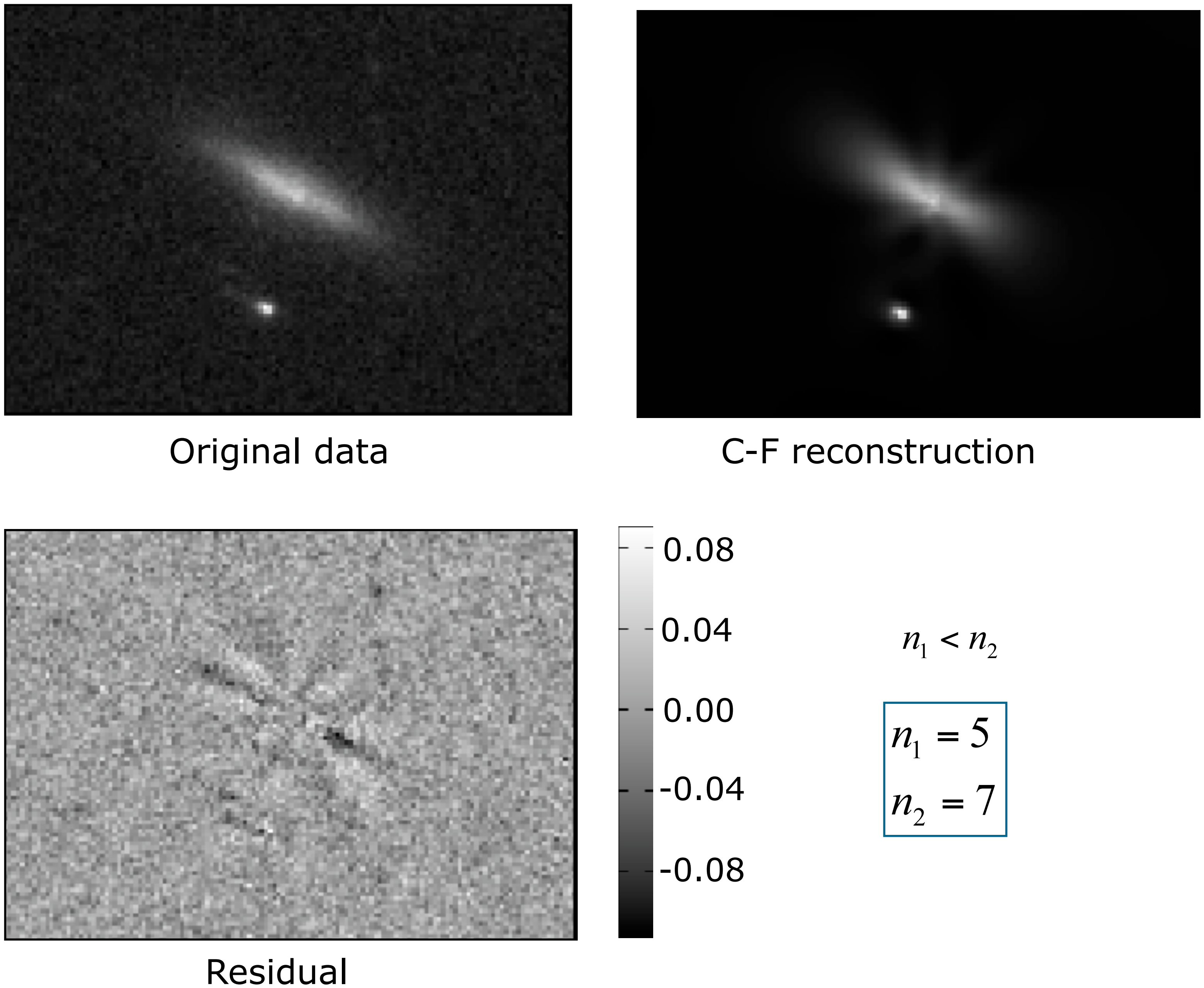
$n_1 = 6, \; n_2 = 1$

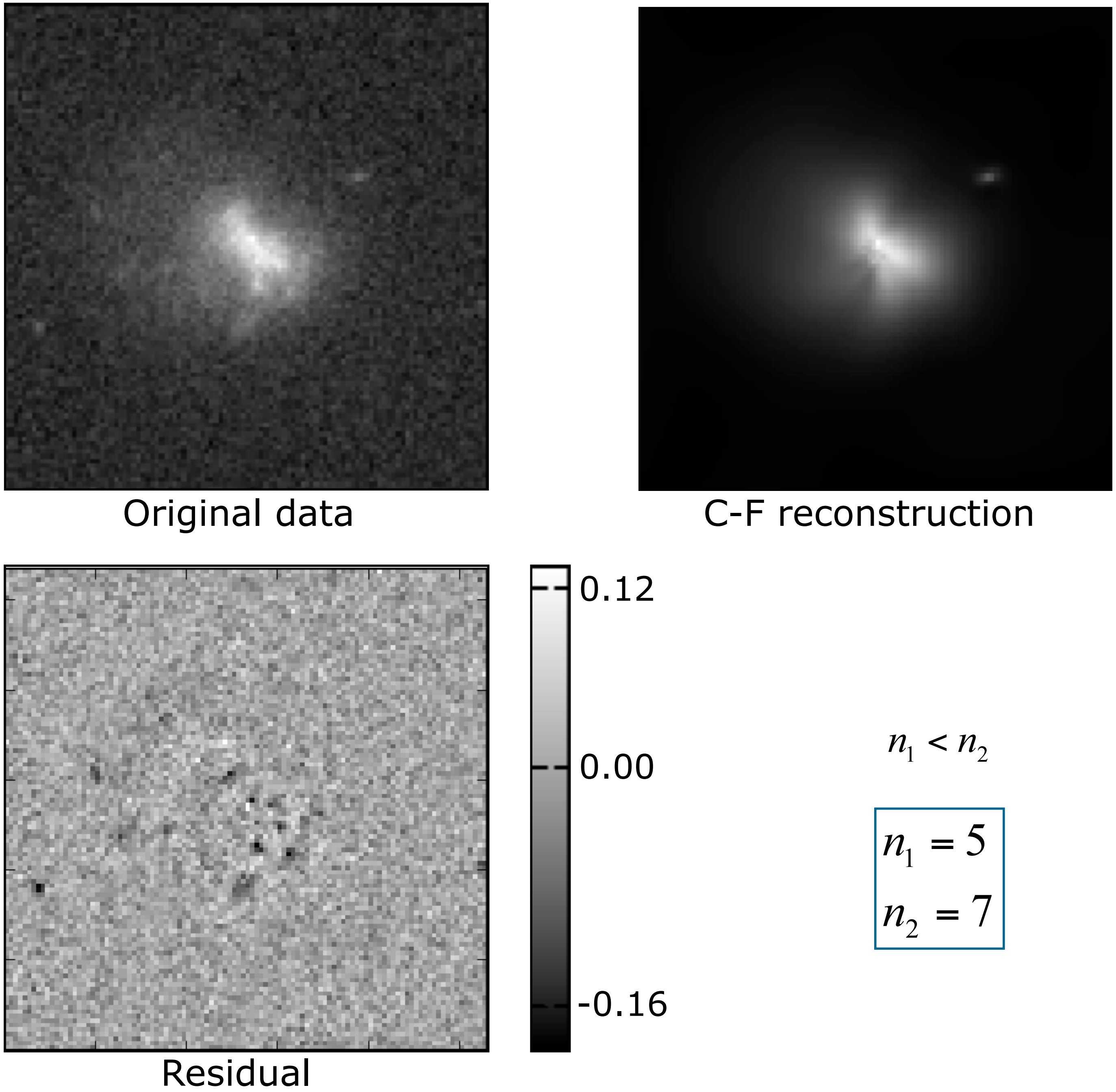


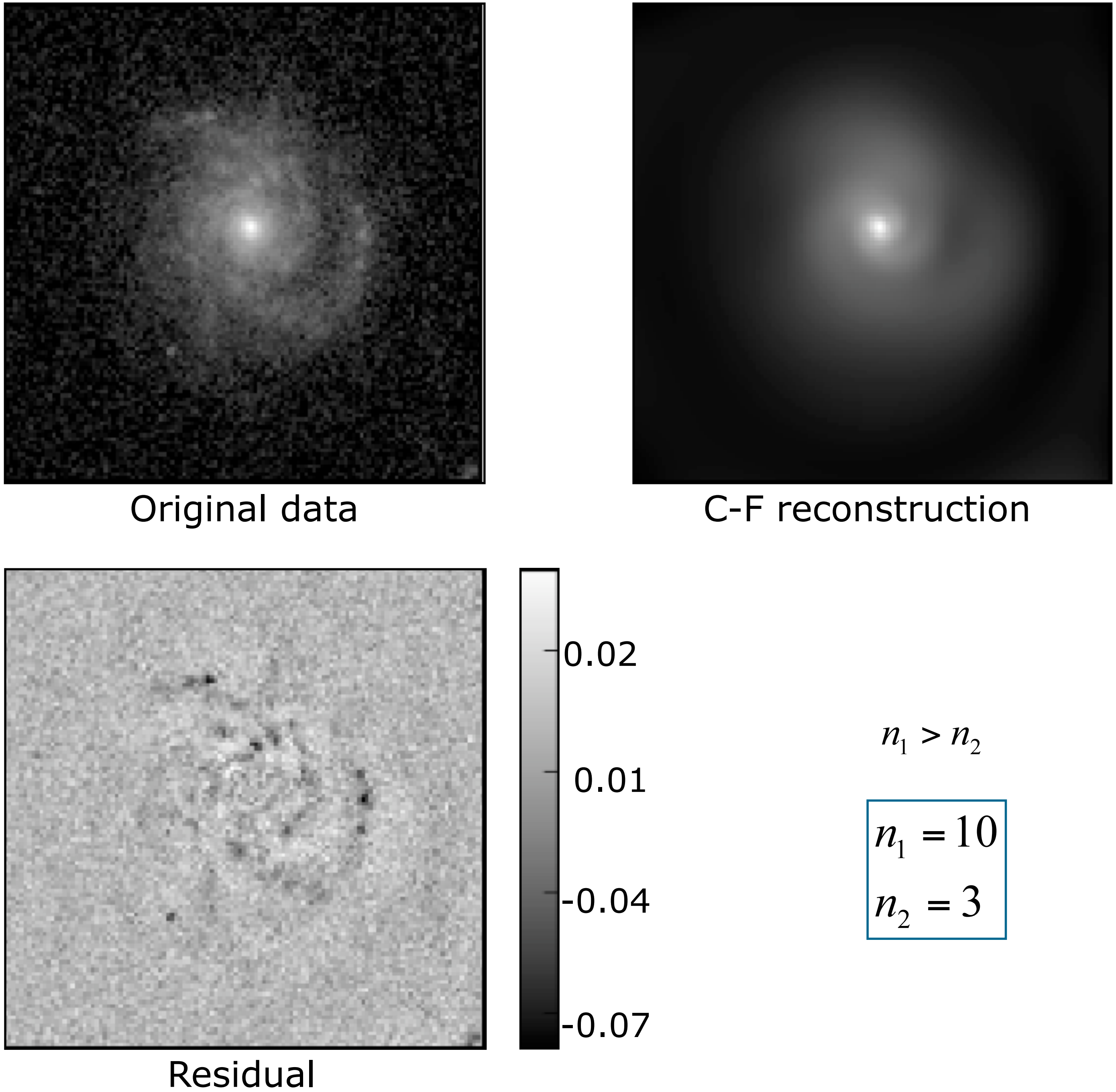
$n_1 = 6, \; n_2 = 2$



$n_1 = 10, \; n_2 = 4$


Original data
C-F reconstruction
Residual
0.08
0.04
0.00
-0.04
-0.08
$n_1 < n_2$
$n_1 = 5$
$n_2 = 7$

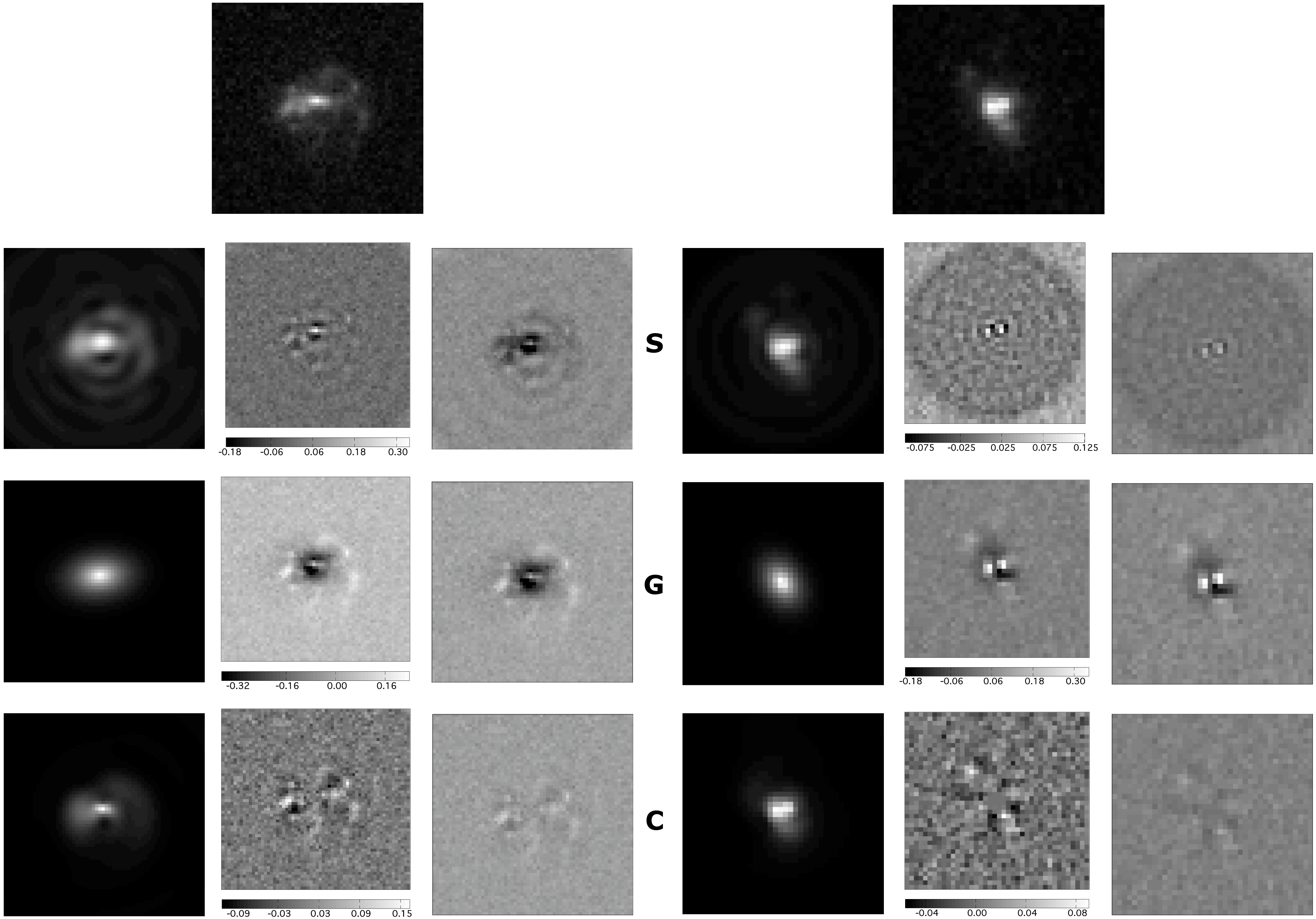

Original data
C-F reconstruction
0.12
0.00
-0.16
Residual
n1 < n2
n1 = 5
n2 = 7



$n_1 > n_2$

$n_1 = 10$

$n_2 = 3$


S
G
C
-0.18 -0.06 0.06 0.18 0.30
-0.075 -0.025 0.025 0.075 0.125
-0.32 -0.16 0.00 0.16
-0.18 -0.06 0.06 0.18 0.30
-0.09 -0.03 0.03 0.09 0.15
-0.04 0.00 0.04 0.08

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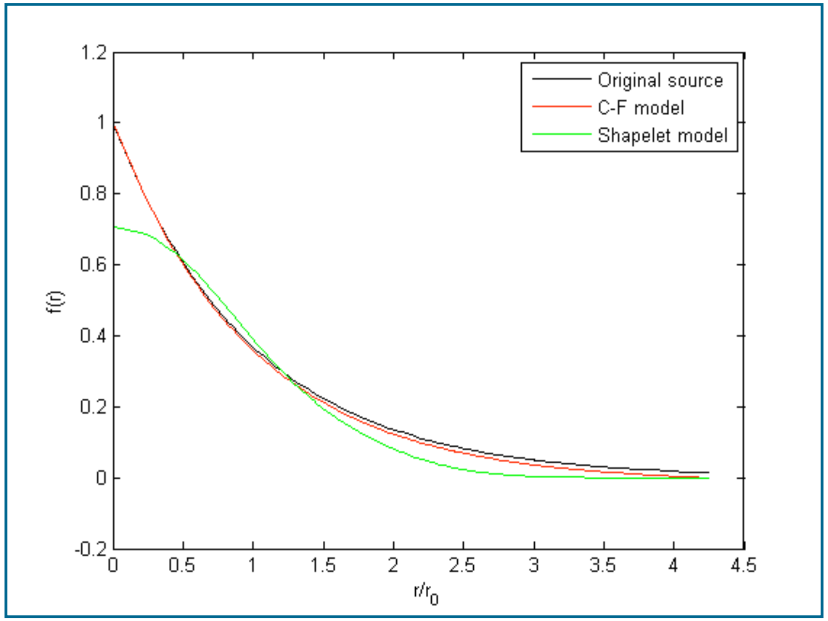
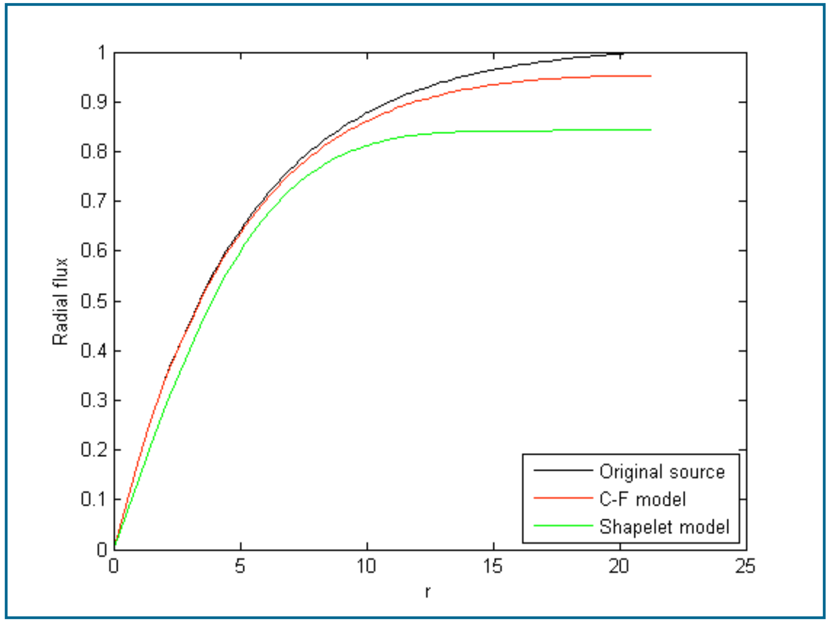


Radial profile

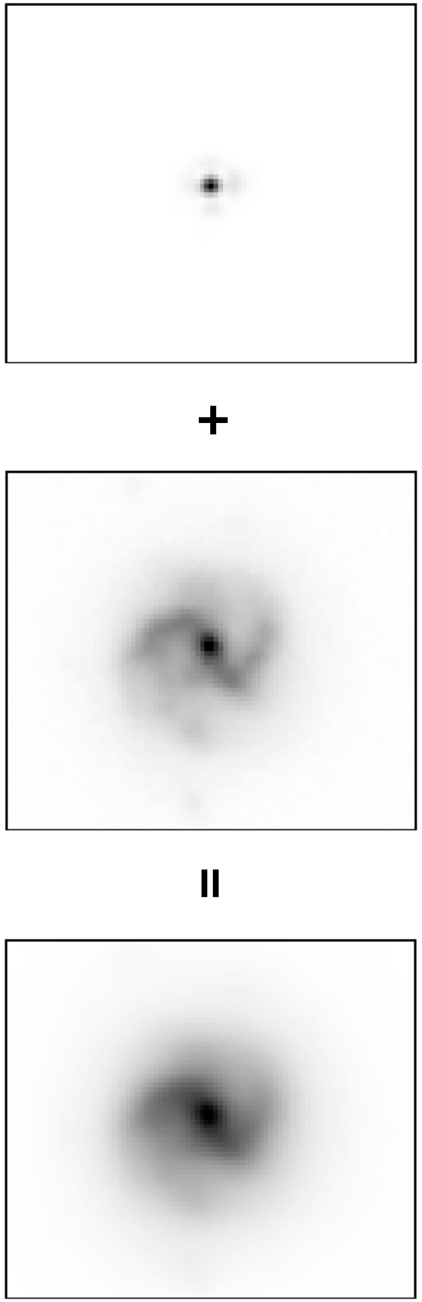


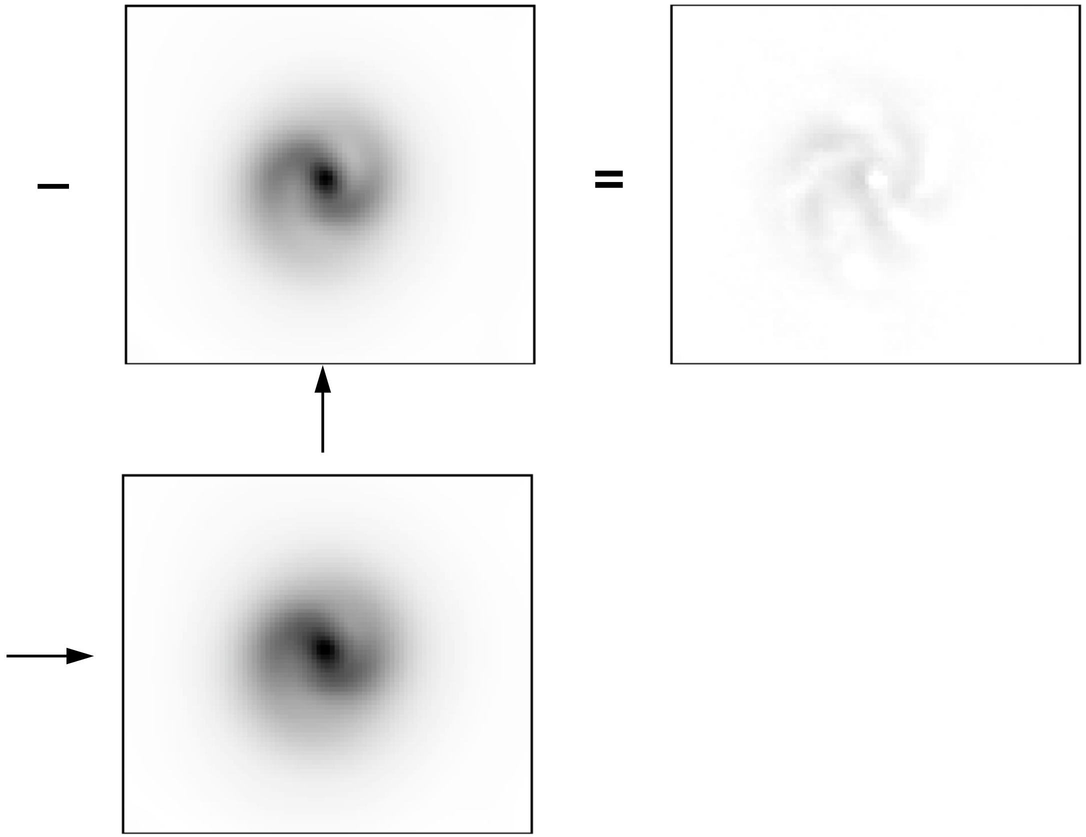
Radial flux

PSF deconvolution

$$f * PSF = \left(\sum_{n_2=-\infty}^{+\infty} \sum_{n_1=0}^{+\infty} f_{n_1 n_2} \phi_{n_1 n_2} \right) * PSF =$$

$$= \sum_{n_2=-\infty}^{+\infty} \sum_{n_1=0}^{+\infty} f_{n_1 n_2} \left(\phi_{n_1 n_2} * PSF \right)$$





Object shape measurement

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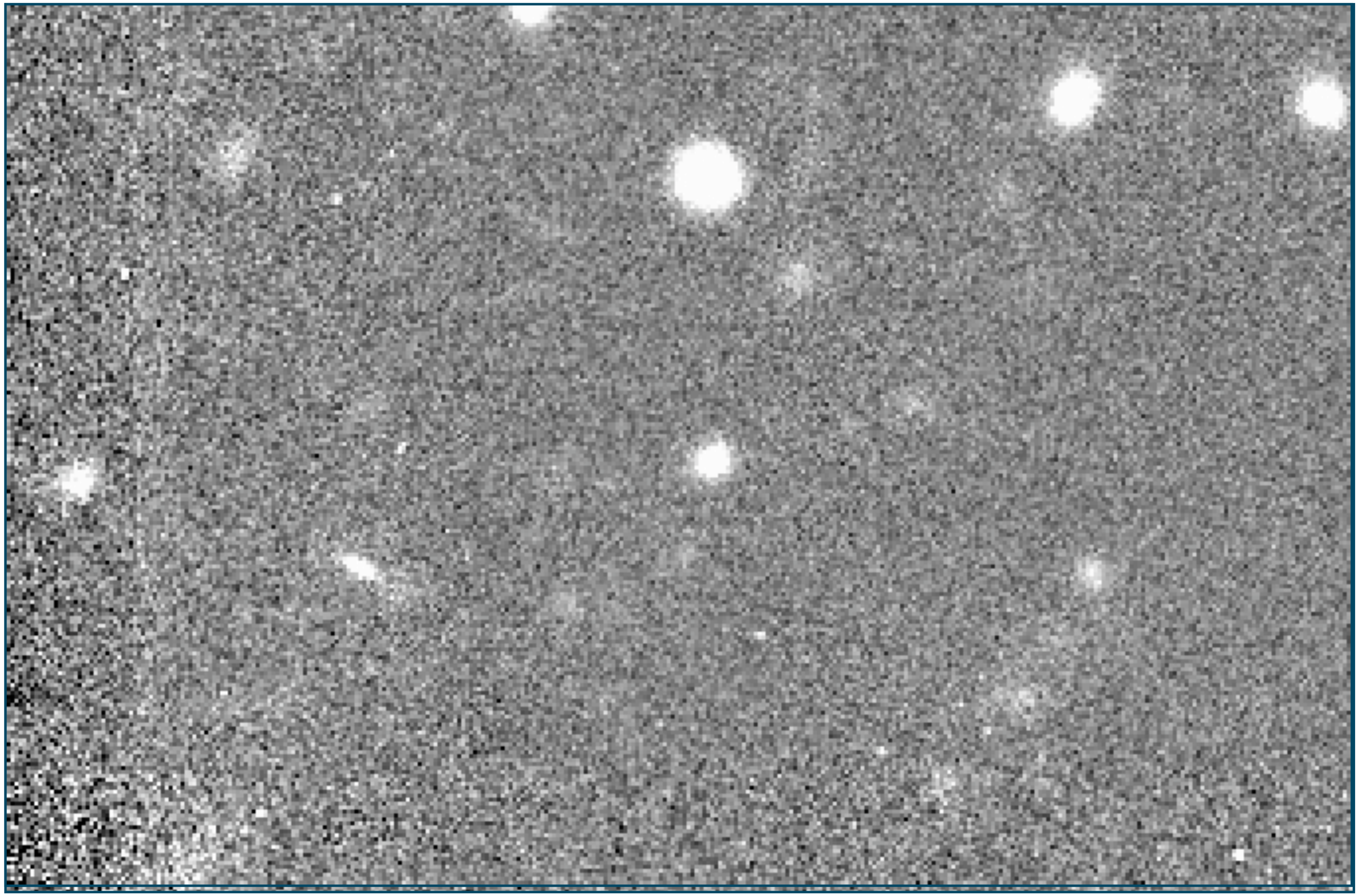
If we define

$$I_p^{n_1} = \begin{cases} 2\sum_{j=0}^{n_1} \binom{n_1}{j} (-1)^j L^{-j/2} \dfrac{R^{p+j/2+1}}{2p+j+2} \mathrm{Re}\left[e^{i n_1 \pi/2} i^{n_1+j} {}_2F_1\left(n_1, 2p+j+2, 2p+j+3; \dfrac{-i\sqrt{R}}{\sqrt{L}} \right) \right], & \text{if } n_1 > 0 \\ \dfrac{R^{p+1}}{p+1}, & \text{if } n_1 = 0 \end{cases}$$

then some morphological parameters can be calculated by means of the C-F coefficients:

- Flux: $F = 2\pi \sum_{n_1=0}^{+\infty} f_{n_1,0} I_1^{n_1}$
- Rms radius: $R^2 = \dfrac{2\pi}{F} \sum_{n_1=0}^{+\infty} f_{n_1,0} I_3^{n_1}$
- Centroid: $x_c + i\, y_c = \dfrac{2\pi}{F} \sum_{n_1=0}^{+\infty} f_{n_1,1} I_2^{n_1}$
- Ellipticity: $\varepsilon = \dfrac{\sum_{n_1=0}^{+\infty} f_{n_1,-2} I_3^{n_1}}{\sum_{n_1=0}^{+\infty} f_{n_1,0} I_3^{n_1}}$

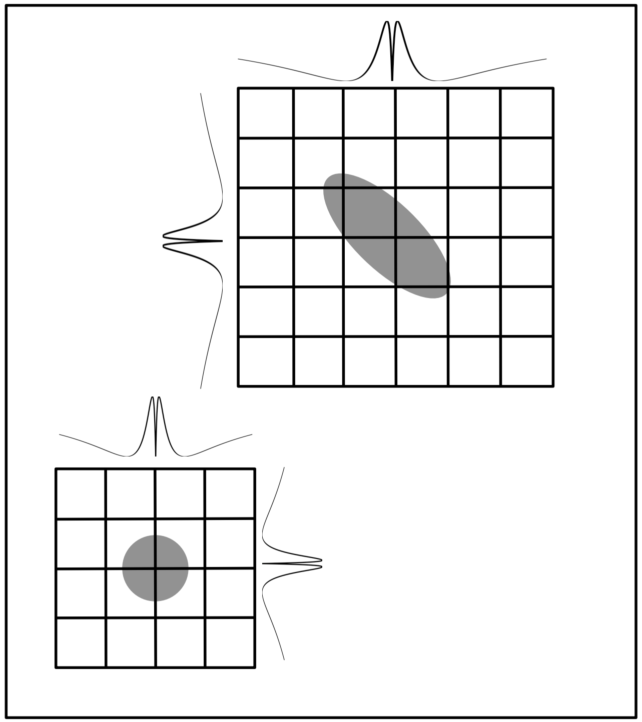
Model adding: example



Clusters processing

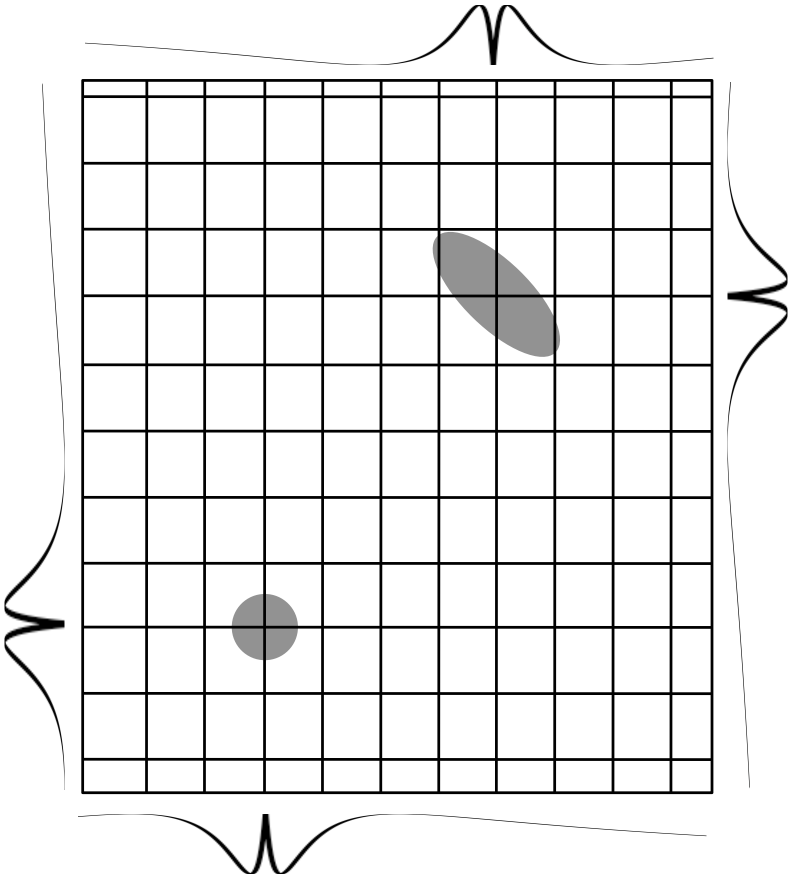
- Method 1

One-by-one processing of the objects, taking different frames.

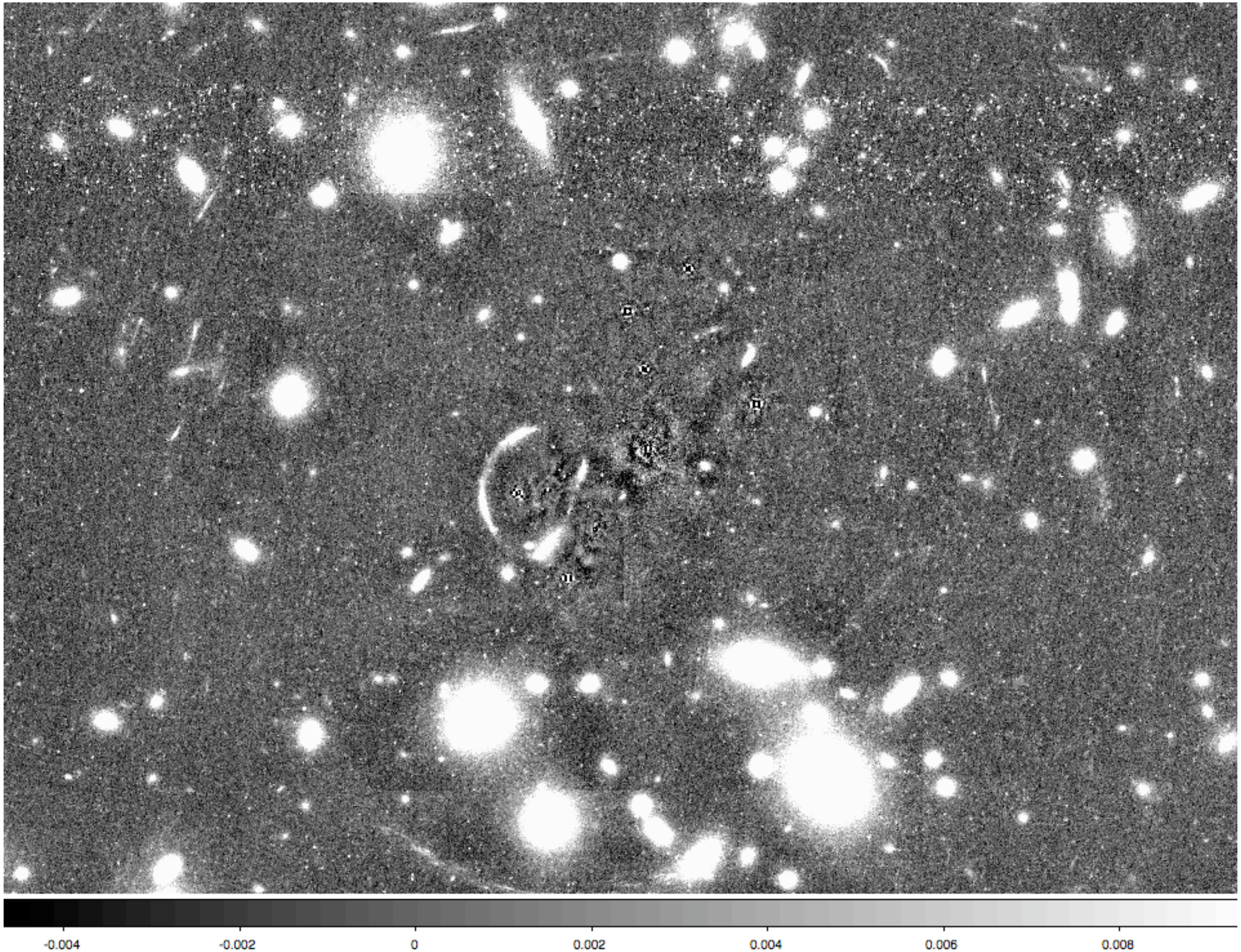


- Method 2

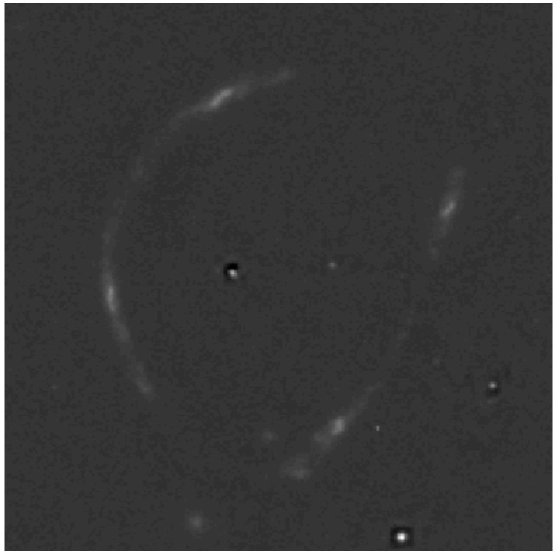
Simultaneous processing of the objects, centering a grid in each object.



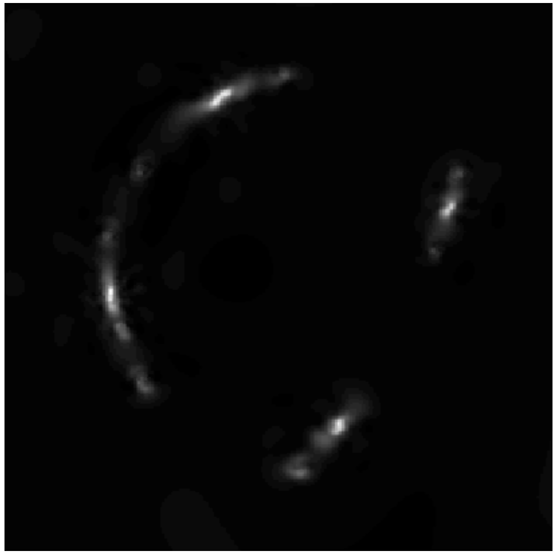
ABELL1703 (arXiv:1004.4660)



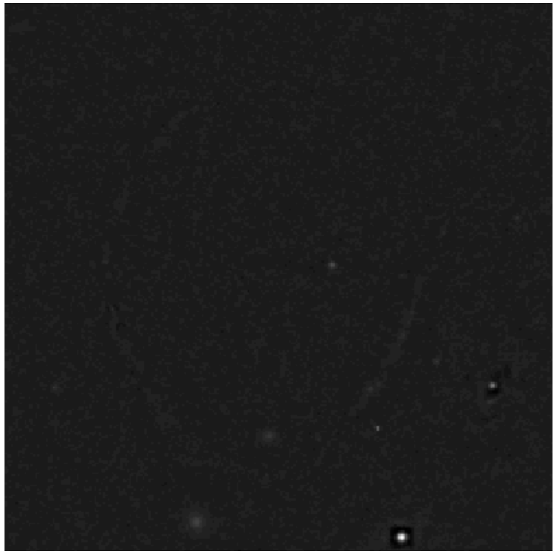
Not only galaxies but also arcs:



Original



Model



Residual

- Cheblet bases have proved to be a highly reliable method to analyze galaxy images, with better results than GALFIT and shapelet techniques.

- Cheblet bases allow us to efficiently reproduce the morphology of the galaxies and measure their photometry.

- PSF deconvolution is easily implemented due to the bases linearity.

- Different morphological parameters can be directly inferred from Cheblet coefficients, with great accuracy.

- Not only single image processing is possible, but also cluster images, just overlapping grids with origin on the different object centers.